Épreuve de section européenne

Chinese remainder theorem

Some early Chinese mathematicians focused on congruence problems and found efficient ways to solve them.

Consider the case of two equations, with the following system, where n_1 and n_2 are coprime:

$$(S)\left\{\begin{array}{lcl} x & \equiv & a_1 (\text{mod } n_1) \\ x & \equiv & a_2 (\text{mod } n_2) \end{array}\right.$$

How to find a solution to this simple system? Since n_1 and n_2 are coprime, meaning that they have no common factor except 1, we know from Bézout's identity that there are some u, $v \in \mathbf{Z}$ that satisfy $n_2 u + n_1 v = 1$. Then a solution is

$$x = a_1 n_2 u + a_2 n_1 v.$$

Adapted from Wikipedia

Questions

1. Are 12 and 9 coprime? Explain why.
2. Read the story below:

   > Rudy stole a pack of candies and must share it with his 5 brothers and 3 sisters. If he divides the pack evenly between his brothers alone, there will be one candy left for him. If he divides the pack evenly between his sisters alone, there will be two candies left for him. What is the minimal number of candies in the pack?

   (a) Express the numerical information contained in this story using congruences.

   (b) Using the fact that $5 \times (-1) + 3 \times 2 = 1$, apply the process described above to solve the problem.
3. Prove that the number x given at the end of the Chinese Remainder Theorem text is solution to the system (S).