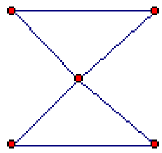
Épreuve de section européenne

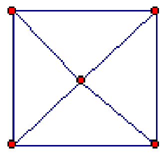
Graph theory

A *graph* is a set of points, called the *vertices* of the graph, that can be connected by line segments, called the *edges* of the graph. The *degree* of a vertex is the number of edges that connect this vertex to other vertices.

An *Eulerian circuit* in a graph, named after Leonhard Euler (pronounced oiler), an 18th century Swiss mathematician, is a circuit, starting and ending at the same vertex, that goes through every edge in the graph exactly once. A graph is *Eulerian* if it has a Eulerian circuit. Euler proved that for a graph to be Eulerian, each vertex must be connected to another one and must have an even degree. Graph A is an example of a Eulerian graph. Graph B is an example of a graph that is not Eulerian.



Graph A



Graph B

The reader is probably familiar with a popular puzzle where one is asked to trace all the lines of a given diagram without picking up the pencil and without retracing any already existing lines. This is precisely equivalent to the problem of finding a Eulerian circuit.

Applications of Eulerian circuits abound. For example, Eulerian circuits are obviously desirable in the deployment of street sweepers, snowplows and mail carriers. In these applications, going down a street more than once is a waste of resources. Thus, Eulerian circuits represent optimal solutions in terms of conserving resources.

Adapted from *An Introductory Graph Theory Curriculum*, by Craig Swinyard

Questions

1. Give the degree of each vertex in graphs A and B.
2. Show a Eulerian circuit in graph A.
3. Why, according to Euler, can't one find a Eulerian circuit in graph B?
4. Modify graph B to make it Eulerian.
5. Can you explain why an odd vertex prevents a graph from being Eulerian?