Épreuve de section européenne

Carlyle's construction for solutions of a quadratic equation

Thomas Carlyle (1795–1881) is best known as a writer but he was also a mathematician. As a writer, Carlyle's success was assured by the publication of his three-volume work "The French Revolution: A History" in 1837. Carlyle started his life as a mathematics teacher.

The following procedure is a construction he found to solve the equation $x^2 + bx + c = 0$ (1) where b and c are real numbers:

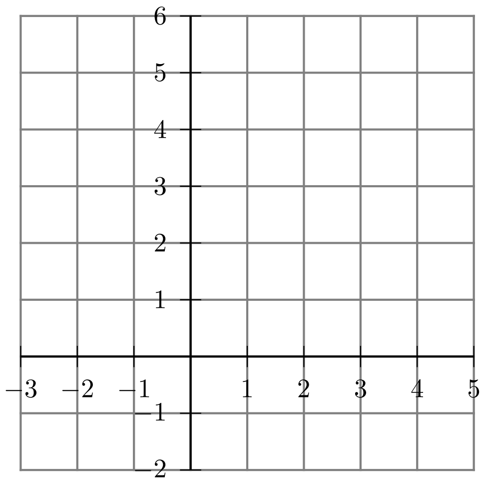
- on graph paper, plot the points $A(0, 1)$ and $B(-b, c)$;
- bisect the line segment AB in M ;
- construct a circle with center M and radius AM ;
- label as P and Q the points where the circle intersects the x-axis.

The directed lengths OP and OQ are the solutions of the equation (1).

Adapted from Allaire and Bradley, *Mathematics Teacher*, Vol. 94, No. 4, April 2001

Questions

1. Let us suppose that $b = -4$ and $c = 3$.
   (a) Following this procedure, construct the circle and the points P and Q.
   (b) Read the solutions of the equation $x^2 - 4x + 3 = 0$.
   (c) Solve algebraically the equation and check the answers obtained by Carlyle's construction.



2. Assume now that $b = 2$ and $c = 4$.
   (a) Construct a second circle and deduce the solutions of the equation $x^2 + 2x + 4 = 0$.
   (b) Check your answer algebraically.
3. We denote r the radius of the constructed circle ($r = AM$) and d the distance between the point M and the x-axis.
   (a) What can we conclude if $d < r$?
   (b) What can we say if $d > r$?
4. Explain in a few words why the construction given by Carlyle is valid.