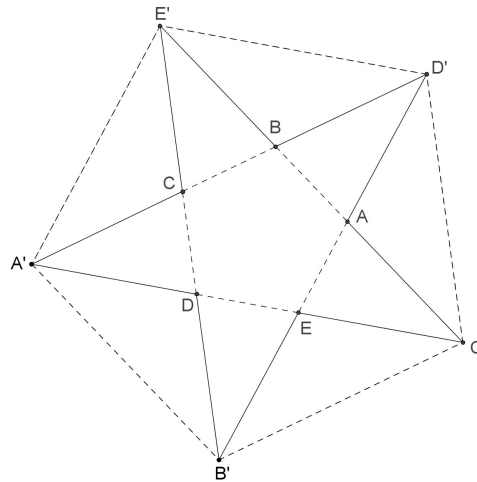## Épreuve de section européenne

The Golden Ratio

In the pentagram $A'DB'EC'AD'BE'C$, which the Pythagoreans regarded as a symbol of health, the ratio $A'B$ to BD' is called the Divine Proportion or Golden Ratio.

Euclid, the Greek mathematician of about 300 BC, wrote *the Elements* which is a collection of thirteen books on Geometry (written in Greek originally).

In Book 6, Proposition 30, Euclid shows how to divide a line according to the golden ratio:
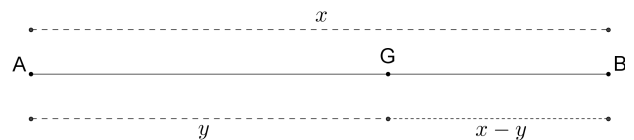
By this construction, *"the ratio of the larger part of this line, AG to the smaller part GB is the same as the ratio of the whole line AB, to the larger part, AG"* (i.e. the ratio AG/GB is the same as the ratio AB/AG).

Throughout *The Elements*, several propositions (theorems in modern terminology) and their proofs employ the golden ratio. Some of these propositions show that the golden ratio is an irrational number. The name "extreme and mean ratio" was the principal term used from the 3rd century BC until about the 18th century.

The modern history of the golden ratio starts with Luca Pacioli's *De Divina Proportione* of 1509, which captured the imagination of artists, architects, scientists, and mystics with the properties, mathematical and otherwise, of the golden ratio. The first known approximation of the (reciprocal) golden ratio by a decimal fraction, stated as "about 0.6180340," was written in 1597 by Michael Maestlin of the University of Tübingen in a letter to his former student Johannes Kepler.

From various sources

Questions

1. Assuming that a regular pentagon is given, explain the construction of a pentagram.

2. a) Where can you find in the text that: $\dfrac{y}{x-y} = \dfrac{x}{y}$?

   b) Let's denote by ϕ the golden ratio, which is equal to $\dfrac{x}{y}$. Prove that we have: $\phi^2 = \phi + 1$.

   c) Find out the exact value of ϕ and give its approximation to 6 decimal places.

3. It is said that the idea of the golden ratio captured the imagination of various artists, architects, scientists (…) throughout history. Can you give examples?