

## Épreuve de section européenne

### Magic squares

Magic squares go back in ancient history, before the Romans, before the Greeks, before the Hebrews, to the ancient Chinese. Over 4,000 years ago, the mythical Emperor Yu is said to have observed some curious markings on the shell of a large tortoise that had emerged from the Lo River. The marks had the shape of a square and when they counted either horizontally, vertically and diagonally they produced the same number (*ie* the magic constant). This was interpreted as a magical and mystical phenomenon and thus the mathematical structure has become known as magic squares. The Lo Shu magic square is known of a magic square of order three: that means the square is made up of three numbers in the horizontal rows and three numbers in the vertical columns. The square is said to be perfect of order  $n$  when it contains all the integers from 1 to  $n^2$ .

|   |   |   |
|---|---|---|
| 4 | 9 | 2 |
| 3 | 5 | 7 |
| 8 | 1 | 6 |

Throughout history up until modern times, magic squares were considered to possess supernatural properties and were used as charms to dispel evil or bring good fortune.

Magic squares made their appearance in Europe through Asia Minor and Constantinople.



In Europe, as in the East, magic squares continued to be associated with the great mysteries of the universe interpreted through astrology. The famous Renaissance sculptor-artist Dürer produced a wood engraving titled *Melencolia I* that includes a magic square of order 4. It features a brooding alchemist in his shop sitting with his head on his hand. He is sullen<sup>1</sup>, all his instruments are in disarray, nothing is being weighed on the balance, nobody is on the ladder and time is running out in his hourglass. On the wall, above the alchemist's head, is the magic square below.

What makes this square unusual is that Dürer has been able to include the year 1514 in the bottom of the square which is the year of the completion of the work. Saturn, the square of order 4, was given sanguine and salutary properties and the artist was using this magic square to bring the alchemist out of his melancholy.

|    |    |    |    |
|----|----|----|----|
| 16 | 3  | 2  | 13 |
| 5  | 10 | 11 | 8  |
| 9  | 6  | 7  | 12 |
| 4  | 15 | 14 | 1  |

From various sources

### Questions

1. Find out the magic constant of Dürer's magic square.
2. Compute the sum of all integers from 1 to 16. Explain why the magic constant of an order 4 perfect magic square is necessarily 34.
3. Name the instrument the alchemist is holding in his right hand. To what purpose is it usually used? What does it symbolize here?
4. Make a perfect magic square of order 3 with all odd numbers from 11 to 27.

<sup>1</sup> Sullen : morose