Épreuve de section européenne

The Gateway Arch

The Gateway Arch in St. Louis, Missouri, was built as a monument to the westward expansion of the United States, and opened to visitors in 1967. Both the width and height of the arch are 630 feet (192 m). The arch is the tallest stainless steel monument in the world. The cross-sections of the arch's legs are equilateral triangles, narrowing from 54 feet (16 m) per side at the bases to 17 feet (5.2 m) per side at the top.

But where does the shape of this arch come from?

In 1675, Robert Hooke, contemporary with Isaac Newton, made the connection between the ideal shape of an arch and that of a hanging chain in an aphorism that says, in abbreviated form, "As hangs the chain, so stands the arch." In other words, the geometry of a standing arch should mirror that of a hanging chain.

In physics and geometry, the catenary (or chaînette) is the curve that an idealized hanging chain or cable assumes under its own weight when supported only at its ends. The curve has a U-like shape, superficially similar in appearance to a parabola (though mathematically quite different).

The catenary curve is the graph of the function f defined by $f(x) = a \times \dfrac{e^{\frac{x}{a}} + e^{-\frac{x}{a}}}{2}$ where a is a positive constant depending on the chain or cable.

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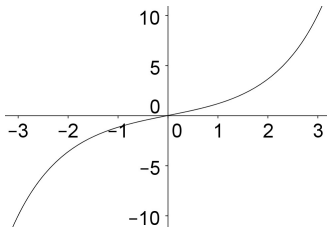
Questions

1. Compute the area of a triangle representing the cross-section at the base of the Gateway Arch.

For the following questions, we will take $a = 1$ for the function f so that $f(x) = \dfrac{e^x + e^{-x}}{2}$, defined on **R**.

2. The graph of the derivative f' of function f is given beside.
Deduce the variation table of function f.



3. We have $f(3) \approx 10$. Find a parabola passing through the points $A(-3;10)$, $B(3;10)$ and $S(0;1)$, then compare that parabola to the graph of function f.

4. Supposing the graph of f is the curve of a hanging chain with x belonging to $[-3;3]$. Find a formula for a function g whose graph corresponds to a standing arch with the same dimensions and with legs lying on the points $A'(-3;0)$ and $B'(3;0)$.