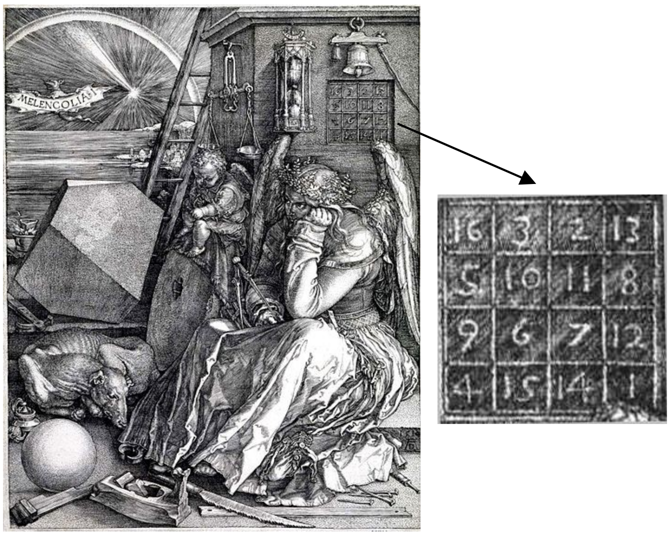
Épreuve de section européenne

Magic squares

On the engraving[1] “Melencolia” of the German Renaissance Master Dürer lies on top right a square with 4 rows and 4 columns. In each cell of this square are the integers from 1 to 16, with the date 1514 of this engraving in the middle of the bottom row.



It is a magic square: the numbers in each row, and in each column, and the numbers that run diagonally in both directions, all add up to the same number. This number is called the magic constant of this magic square.

If n (a natural number) stands for the number of rows (and columns), the square is described as being of order n. And the square is said normal if it contains all the integers from 1 to n^2.

Thus, Dürer’s square is a normal magic square of order 4.

A normal magic square of order n has always for magic constant the number $\frac{n(n^2+1)}{2}$. It is possible to construct a normal magic square of any order n, except for $n=2$.

In 1688, the French diplomat De La Loubère described a method to construct a normal magic square of any order n when n is odd:

Starting from the central box of the first row with the number 1, the fundamental movement for filling the boxes with the next number of the list 2, 3, … , is n² diagonally up and right one step at a time. When a move leaves the square, it is wrapped around to the last row or first column, respectively. If a filled box is encountered, one moves vertically down one box instead, then continuing as before.

This method is explained with $n=3$ as below:

	1	

step 1

	1	
		2

step 2

	1	
3		
		2

step 3

	1	
3		
4		2

step 4

	1	
3	5	
4		2

step 5

	1	6
3	5	
4		2

step 6

and so on …

Adapted from many sources, including *“Wikipedia”*

[1] engraving = gravure

Questions

1. a) What is the magic constant of Dürer's magic square?

   b) Compute the magic constant for a normal magic square of order 3, and the one for a normal magic square of order 5.

2. Complete the method of De La Loubère for the normal magic square of order 3.

3. In the same way, construct a normal magic square of order 5 (you can use the empty squares at the end).

4. Explain why the construction of a normal magic square of order 2 is impossible.

5. We admit that the first natural numbers from 1 to p add up to $\frac{p(p+1)}{2}$. Prove that the magic constant of a normal magic square of order n is $\frac{n(n^2+1)}{2}$.

