

Épreuve de section européenne

Twin primes

Prime numbers are a rich and ancient source of mathematical mystery. It has been known for over 2000 years that there is an infinite number of them.

Let us study some pairs of prime numbers called twin primes. Two prime numbers are twin primes if they differ by 2. If you look at a list of the first 50 primes you'll see that it contains 16 twin prime pairs.

The question is: are there infinitely many? A more general version of this question was put forward in 1850 by Alphonse de Polignac. He postulated that there were infinitely many consecutive primes separated by 2 (twin primes), 4, 6, etc. and by every even number.

Twin primes are used to define one of mathematics' more unusual constants. In 1919 Viggo Brun proved that the sum of the reciprocals of all the twin primes converged. The resulting constant, B , is known as Brun's constant in honour of this achievement. Despite this proof it is still not known how many twin primes there are!

$$B = \left(\frac{1}{3} + \frac{1}{5}\right) + \left(\frac{1}{5} + \frac{1}{7}\right) + \left(\frac{1}{11} + \frac{1}{13}\right) + \dots$$

From plus.maths.org

Questions

1. Give the definition of a prime number.
2.
 - a. Give the prime numbers that are under 30 (there are 10 of them).
 - b. Can you find twin primes among them?
3.
 - a. Prove that 197 is prime, detailing your method.
 - b. Prove that 197 is part of a twin prime pair.
4. Let p_n and $p_n + 2$ be the members of the n^{th} pair of twin primes.
 - a. Give p_1 , p_2 and p_3
 - b. Let $u_n = \sum_{i=1}^n \left(\frac{1}{p_i} + \frac{1}{p_i + 2} \right)$. Give u_1 , u_2 and u_3 .
 - c. Prove that (u_n) increases.
 - d. What other information would we need to prove that (u_n) converges?