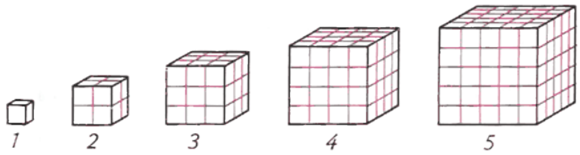
Épreuve de section européenne

The sum of the first cubes



Students often encounter formulas for sums of powers of the first *n* positive integers as examples of statements that can be proved using the Principle of Mathematical Induction. Formulas for sums of integer powers were first given in generalizable form in the West by Thomas Harriot (c. 1560-1621) of England. Pierre de Fermat (1601-1665) is often credited with the discovery of formulas for sums of integer powers, but his fellow French mathematician Blaise Pascal (1623-1662) gave the formulas much more explicitly. Before them, Greek mathematicians Pythagoras and Archimedes gave famous formulas for some sums. A few centuries after them, the northern Indian mathematician and astronomer, Aryabhata, born in 476, wrote one of the earliest known Indian mathematics and astronomy books, the *Aryabhatiya.* In Section II, he wrote:

"*The sum of the first numbers is the half part of the product of number of terms and the number of terms plus one. The square of the sum of the first numbers is the sum of the cubes.* "

In our notation, it says that:

$$1+2+3+...+n=\frac{n(n+1)}{2} \quad \text{and} \quad 1^3+2^3+3^3+...+n^3=(1+2+3+...+n)^2$$

Hence, the formula for the sum of the cubes is :

$$1^3+2^3+3^3+...+n^3=\frac{n^2(n+1)^2}{4} \qquad (P)$$

Part from an article written by Janet Beery (University of Redlands), 2012

Questions

1) Give the name of the Greek and French mathematicians who worked on sums of integers.
2) What century was the *Aryabhatiya* book probably written in?
3) Is the sequence $1^3;\ 2^3;\ 3^3;\ 4^3...$ an arithmetic one? A geometric one?
4) Prove that (*P*) is true for $n=1,\ n=2,\ n=3$.
5) Prove (*P*) by induction.