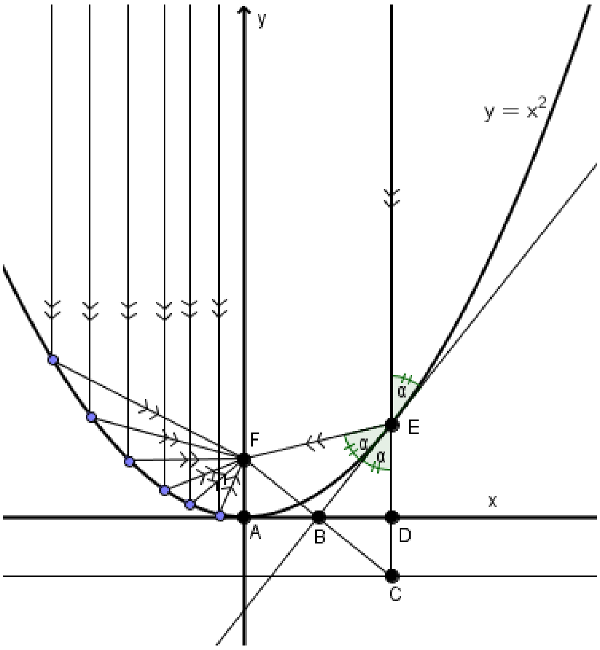
Épreuve de section européenne

The reflective property of a parabola

The reflective property of a parabola states that, if a parabola can reflect light, then light which enters it parallel to the axis of symmetry is reflected to the focus.

An ancient tale describes the Greek inventor Archimedes, using that characteristic of the parabola, to set fire to Roman ships attacking his home city of Syracuse in 212 BC. Let us prove that property.



Consider the parabola $y = x^2$ in an orthonormal basis.

The point $\mathrm{E}(x; x^2)$ is an arbitrary point on the parabola.

The focus is $\mathrm{F}(0; \frac{1}{4})$

The vertex is $\mathrm{A}(0;0)$

The line FA (the y-axis) is the axis of symmetry.

The parallel to FA passing through E intersects the x-axis at $\mathrm{D}(x;0)$

The line of equation $y = -\frac{1}{4}$ (called the directrix of the parabola) contains the point $\mathrm{C}(x; -\frac{1}{4})$

The point B is the midpoint of the line segment FC.

Adapted from http://en.wikipedia.org/wiki/Parabola

Questions

1. Prove that the coordinates of B are $(\frac{x}{2};0)$.
2. Prove that the slope of the line BE is equal to $2x$.
3. Why can we conclude that BE is tangent to the parabola at E ?
4. Prove that $\overrightarrow{\mathrm{FC}}.\overrightarrow{\mathrm{BE}} = 0$
5. Deduce that EFC is an isosceles triangle.
   Prove that the angles marked α are equal (This means that a ray of light which enters the parabola and arrives at E parallel to the axis of symmetry will be reflected by the line EB so it travels along the line EF, as shown in the diagram).