Épreuve de section européenne

A Mersennery Quest

Almost all the largest primes found in recent years are of a particular form $M(n) = 2^n - 1$. They are called Mersenne primes.

The search for the largest prime will never end. Euclid proved that the number of primes is unlimited. His argument is a model of logical simplicity. Suppose there are only a finite number of primes, and let *p* be the largest of them. Form the number $N = p! + 1$. Clearly, *N* is not divisible by any number up to *p*, because it always leaves a remainder of 1. So either *N* is divisible by some prime number greater than *p* or it is itself prime. In either case, there is a prime number greater than *p*.

But there are arbitrarily large gaps between primes. This is simply shown by construction. For any *n*, take the sequence $\{ n!+2, n!+3, n!+4, \dots , n!+n\}$; the first number is divisible by 2, the second by 3 and so on, so the sequence comprises *n-1* consecutive composite numbers.

Mathematicians have long searched for functions that generate only prime values. Euler found a nice quadratic polynomial $f(n) = n^2 - n + 41$ that is prime for the first 40 values of *n*.

Christian Goldbach, a friend of Euler, also stated that every even number (greater than 2) is the sum of two primes. This conjecture, called Goldbach's conjecture, is another result crying out for a proof. Fame awaits anyone who can prove it.

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Questions

1. Explain what prime numbers and composite numbers are.

2. Give the first five Mersenne primes.

3. What kind of problems in mathematics use prime numbers?

4. The text says that *f(n)* for *n=0* to *n=40* is prime. Is it also true of *f(41)*?

5. Check Goldbach's conjecture with two even numbers of your own choice.

6. Do you know other conjectures in maths?