Épreuve de section européenne

Financial discounting

A standard home mortgage[1] works like this : a prospective home owner borrows[2] a sum of money to buy a house at some interest rate, with equal periodic payments (usually monthly) for a specific length of time. The house serves as collateral for the loan[3] and thus is "mortgaged".

The algebra of compounding –calculating the future value of some present value that grows at some compound interest rate or growth rate- provides a helpful introduction to the algebra of mortgages:

$$V_0(1+r)^n = V_n \qquad (1)$$

Here V_0 is the present value, r is the growth rate per period, n is the number of periods that the value is compounded, and V_n is the value in n periods.

If V_0 is the unknown, the process is discounting (a future value V_n is discounted to the present value V_0 at r percent per period for n periods), we have the following:

$$V_0 = \frac{V_n}{(1+r)^n} \qquad (2)$$

Now let's suppose that a buyer borrows V_0 to purchase a house. If the monthly mortgage payments M are equal, the equation (2) can be expanded to calculate the value of the home mortgage (i.e the principal amount borrowed to purchase the house).

We can express V_0 in terms of the monthly payment M, monthly fixed interest rate r, and number of months N the loan runs, by the following:

$$V_0 = \frac{M}{(1+r)^1} + \frac{M}{(1+r)^2} + \ldots + \frac{M}{(1+r)^{N-1}} + \frac{M}{(1+r)^N} \qquad (3)$$

Hence, if M is the unknown: $$M = V_0 \frac{r}{1-(1+r)^{-N}} \qquad (4)$$

Adapted from NCTM

[*mortgage* : hypothèque]
[*to borrow* : emprunter]
[*loan* : prêt]

Questions

1. What future value does an amount of \$100 deposited in a bank account and earning 6% a year (compounded annually) produce in 10 years?
2. What is the present value of a future \$1,000 twenty years from now discounted at 10% per year?
3. For a 30-year fixed-rate mortgage of \$200,000 (i.e for equal monthly payments over 360 months) at 0.5 % per month, show that the equal monthly payment M is close to 1,200\$.
4. Explain how to get formula (4) from formula (3).