Épreuve de section européenne

How to compute decimal logarithms

John Napier is a British mathematician considered as the inventor of logarithms. The logarithm to base 10 of a number x is the only real number y such that $10^y = x$. It is usually noted $\log x$. For example, as $10000 = 10^4$, $\log 10000 = 4$.

Napier set out to find methods to compute the logarithm of a real number, and experimented with some half a dozen before hitting one that was as quick as it was accurate. To understand it, we must recall two different kinds of averages, the arithmetic mean and the geometric mean. The one we use most, the ordinary 'average' of everyday affairs, is the arithmetic mean. The arithmetic mean of any two numbers (say 4 and 16), is found by adding them and dividing the answer by 2. (Thus 4 and 16 makes 20; divide by two, the answer is 10. The arithmetic mean of 4 and 16 is 10.) The geometric mean is used for scientific purposes. To find it, you multiply the two numbers (say 4 and 16, again) and take the square root of the result. (Thus, 4×16 is 64; the square root of 64 is 8; 8 is the geometric mean of 4 and 16.)

Remembering that a logarithm is a power, we can state the following rule for any two numbers whose logarithms we know: the logarithm is the arithmetic mean of the original two logarithms; the number is their geometric mean. For example (working in base 10), let us take the numbers 100 and 1000. To what power do we have to raise the base 10 to get the number 100? What power for the number 1000? Answers: to the first question, 2; to the second, 3. The rule says: the unknown number is the geometric mean of 100 and 1000 (that is, the square root of 100×1000, that is, 316.226677). The logarithm is the arithmetic mean of 2 and 3 ($2+3$ divided by $2 = 2.5$). In other words, the logarithm to base 10 of 316.226677 is 2.5.

Adapted from *Number*, John McLeish, 1991

Questions

1. Explain why the logarithm to base 10 of 100 and 1000 are respectively 2 and 3.
2. Give an example of the use of an average in everyday life.
3. (a) Knowing that for any non-negative real numbers x and y, $\log(xy) = \log x + \log y$, prove that for any non-negative real number x, $\log(\sqrt{x}) = \frac{1}{2}\log x$.

   (b) Translate the rule 'The logarithm is the arithmetic mean of the original two logarithms; the number is their geometric mean.' with a formula and prove it.
4. Apply the method to find the number whose logarithm is 3.5.