Épreuve de section européenne

Mersenne Numbers

Many mathematicians and hobbyists search for large prime numbers. As of January 2014, the largest known prime number is $2^{57,885,161}-1$, a number with 17,425,170 digits. Many of the largest known primes are Mersenne primes.

The Mersenne numbers are numbers of the form 2^n-1. These numbers have fascinated number theorists, particularly because of their connection with perfect numbers: numbers that are the sum of their divisors, including 1 but not the number itself (6, 28, 496, ...). If $p=2^n-1$ is a Mersenne prime, it automatically leads to a perfect number by way of Euclid's formula $2^{n-1}\times p$.

It is easy to show that a Mersenne number cannot be prime unless the exponent *n* is prime. If *n* is prime, will the Mersenne number be prime? The strong law of small numbers suggests it will, because it is true when *n* equals 2, 3, 5, and 7. The law fails for $n=11$, however, because $2^{11}-1$ equals 2047, which equals 23×89. It holds for $n=13$, $n=17$ and $n=19$, then fails again for $n=23$. From here on, successes rapidly become rarer. At the moment only 48 Mersenne primes (hence only 48 perfect numbers) are known.

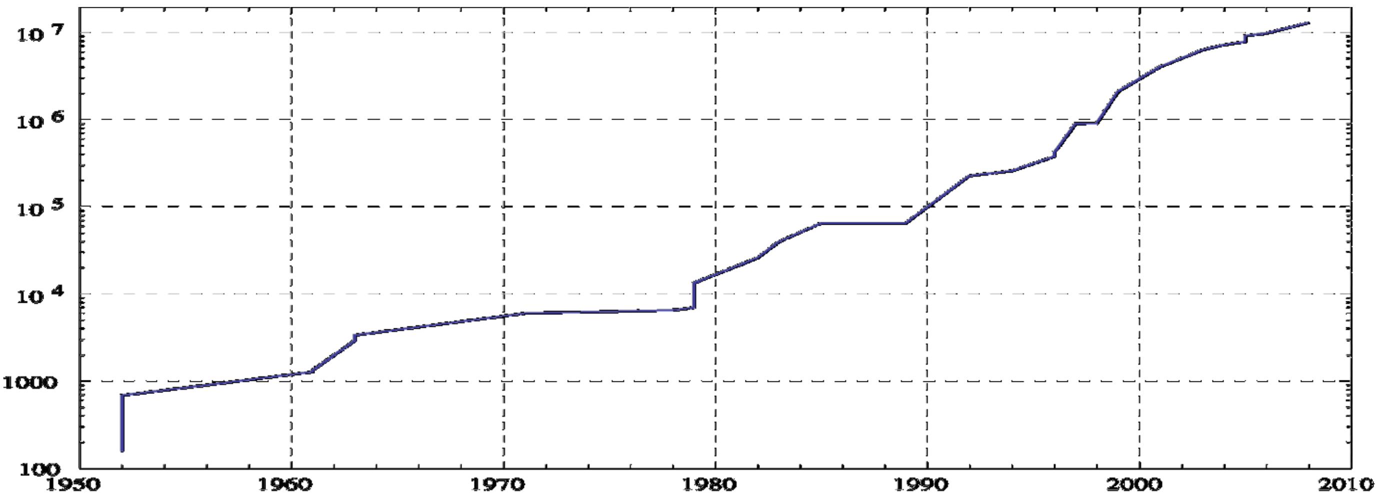

Figure 1: Graph of number of digits in largest known prime by year, since the electronic computer's invention. Note that the vertical scale is logarithmic.

From *The Last Recreations*, by Martin Gardner, and from Wikipedia

Questions

1. Give two interests of Mersenne numbers.

2. Is every Mersenne number a prime?

3. "The strong law of small numbers" is a humorous title of a popular paper by a mathematician. What could that law refer to in the text? What do you think of that law?

4. Describe the evolution of the number of digits in largest known primes since 1950. How would you qualify it?

5. How can you build a perfect number out of a Mersenne number? Is it true for every Mersenne number? Can you prove it?