Épreuve de section européenne

Diophantus's method for Pythagorean Triples

A Pythagorean triple consists of three positive integer *a*, *b*, and *c*, such that $a^2 + b^2 = c^2$. Diophantus's method to find 'all ' Pythagorean triples is the following:.

Take any two whole numbers that we call 'generator numbers', and form:

- twice their product
- the difference between their squares
- the sum of their squares

Then the resulting three numbers are the sides of a Pythagorean triangle.

If we take numbers 3 and 2, then

- twice their product = $2 \times 3 \times 2 =$ 12
- the difference between their squares = $3^2 - 2^2 = 5$
- the sum of their squares = $3^2 + 2^2 = 13$

and we get the "famous" 5-12-13 triple.

But numbers 42 and 23, on the other hand, lead to the 1 235 - 1 932 - 2 293 triangle and no one has ever heard of it .

There's a final twist to Diophantus' rule. Having worked out the three numbers of a triple, we can choose any other number we like and multiply this triple all by the number chosen. So the 5-12-13 triangle can be converted into a 15-36-39 triple by multiplying all three numbers by 3. We can't get these two triples from the above prescription using whole numbers. Diophantus knew that.

Adapted from *Cabinet of mathematical curiosities* by Ian Stewart

Questions

1) Check that 5-12-13 is a Pythagorean triple

2) Find another 'famous' Pythagorean triple starting Diophantus' method with 2 and 1.

3) Does the Diophantus' method give all the Pythagorean triples ?

4) Is 20-21-25 a Pythagorean triple? Find the 2 generator numbers of this triple.