Épreuve de Section Européenne

Descartes's rule of Signs

A number r is a root of a polynomial P if it is a solution of the equation $P(x)=0$. For example, the roots of the polynomial $2x^2+3x-2$ are -2 and $\frac{1}{2}$ because these numbers are the two solutions of the equation $2x^2+3x-2=0$.

Descartes' s rule of signs, first described by René Descartes in his work *La Géométrie*, is a technique for determining the number of positive or negative real roots of a polynomial. The rule for positive roots is as follow:

> The number of positive roots of a polynomial with real coefficients is equal to the number of 'changes of sign' in the list of its ordered non zero coefficients, or is less than this number by a multiple of 2. That is to say : if the number of changes of sign of a polynomial is n, then the number of positive roots is n, or $n-2$, or $n-4$, etc.

Example 1 :
The polynomial $-x^3+2x^2+5x-6$ has only two sign changes (between the first and the second term, then between the third term and the final term), so Descartes' s rule of Signs indicates that the polynomial has two or zero positive roots. Indeed, it has two positive roots (1 and 3), and also a negative one -2.

Example 2 :
The polynomial $x^5-11x^4+33x^3+11x^2-154x+120$ has four sign changes, so Descartes' Rule of Signs indicates that the polynomial has four, two or zero positive roots. Indeed, it has five roots (which are –2, 1, 3, 4 and 5).

From various sources

Questions

1. Solve the equation $2x^2+3x-2=0$.
2. How would you process to check that -2 is a root of the polynomial $-x^3+2x^2+5x-6$?
3. According to the Descartes' s rule of Signs, how many positive roots could have the polynomial $7x^6-5x^5+3x^4+2x^3-3x^2+x-2$? Explain the steps of your answer.
4. Is it possible for the polynomial x^3+4x^2-5x-6 to have two positive roots ?