Épreuve de section européenne

Fun tricks with numbers

-   Hey Jenny ! Here's a fun trick I want to show you, Karl. I'm going to guess the month and the day of your birthday. Ok, First, take the month number from your birthday (January = 1, February = 2 etc.)
-   Ok! Yes ! I have this number.
-   Multiply it by 5. Add 6. Multiply that total by 4. Add 9. Multiply this total by 5 once again. Finally, add to that total the day you were born on. What's your number ?
-   It's 375.
-   Ok, you were born on the 10^{Th}, February ! I just have to subtract 165 from 375 ! I find then 2 for the month (February) and 10 for the day !
-   Yes! It's true !
-   I like these fun tricks. I also know how to check if an integer is divisible by 2, by 5, by 10, by 7, by 13, by 17, by 19…
-   Yes it's easy for the first ones, but I don't know the rules for 7, 13, 17 or 19…
-   For 7 it's an algorithm called L-2M. What you do is to double the last digit of the number X and subtract it from X without its last digit. For instance, if the number X you are testing is 24689, you would subtract 18 from 2468. Repeat this procedure until you get a number that you know for sure it is or it is not divisible by seven. Then the X's divisibility will be the same.
-   Ok ! I check… 24689, 2450, 245 and 14 ! So 24689 is divisible by 7.
-   Yes, and for 13 the rule is called L+4M! For 17, it's L-5M, and for 19 it's L+2M !
-   Great !

Adapted from http://www.lifesmith.com/mathfun.html

Questions

a) Check the birthday fun trick your dates (with your calculator if necessary).
b) Let M be the month number and D the day number. Show that the expression after the steps of the calculation is $100 M + D + 165$. Then explain the trick.
c) Give the ways to check if an integer is divisible by 2, by 5 and by 10.
d) Check the L-2M algorithm with 552671 and 87949.
e) Use the algorithms to show that 20995 is divisible by 13, by 17 and by 19.