Épreuve de Section Européenne

Goldbach's ternary conjecture

In may 2013, H. A. Helfgott claimed to have proven one of the oldest and well-known unsolved problems in number theory. The following text is his introduction :

The ternary Goldbach conjecture (or three-prime problem) states that every odd number *n* greater than 5 can be written as the sum of three primes. Both the ternary Goldbach conjecture and the (stronger) binary Goldbach conjecture (stating that every even number greater than 2 can be written as the sum of two primes) have their origin in the correspondence between Euler and Goldbach (1742).

Vinogradov showed in 1937 that the ternary Goldbach conjecture is true for all *n* above a large constant *C*. Unfortunately, while the value of C has been improved several times since then, it has always remained much too large ($C \approx e^{3100} \approx 2\times10^{1346}$) for a mechanical verification up to *C* to be even remotely feasible. The situation was paradoxical: the conjecture was known above an explicit *C*, but, even after seventy years of improvements, this *C* was so large that it could not be said that the problem could be attacked by any conceivable computational means within our physical universe. Thus, the only way forward was a series of drastic improvements in the mathematical, rather than computational, side.

The present paper proves the ternary Goldbach conjecture. The proof given here works for all *n* greater than $C = 10^{27}$. Verifying the main theorem for *n* less than 10^{27} is really a minor computational task ; it was already done for all *n* less than 8.875×10^{30}.

From *The Ternary Goldbach Conjecture is true*, H. A. HELFGOTT
http://arxiv.org/abs/1312.7748

Questions

1. a) What exactly did H. A. Helfgott claim to have proven in his paper ?
   b) Why is it enough to deduce the ternary Goldbach conjecture ?
2. Explain the binary Goldbach conjecture, and the ternary Goldbach conjecture. Give the binary decomposition of 24, and the ternary decomposition of 27.
3. The binary conjecture is called "stronger" because if it is proven, then we can deduce a proof for the ternary conjecture. Let us imagine that the binary conjecture is true.
   a) If *n* is an odd integer greater than 5, what can you deduce about the integer $n-3$?
   b) Prove that *n* is the sum of three primes.