Épreuve de section européenne

MU puzzle

Suppose there are the symbols M, I, and U which can be combined to produce strings of symbols called *words*. The MU puzzle asks one to start with the *axiomatic* word MI and transform it into the word MU using in each step one of the following transformation rules:

1. Add a U to the end of any string ending with an I. For example: MI to MIU.
2. Double the string after the M (that is, change Mx, to Mxx). For example: MIU to MIUIU.
3. Replace any III with a U. For example: MUIIIU to MUUU.
4. Remove any UU. For example: MUUU to MU.

The transformation rules can be written in a more schematic way. Suppose x and y behave as variables (standing for strings of symbols). Then the transformation rules can be written as:

1. xI → xIU
2. Mx → Mxx
3. xIIIy → ?
4. xUUy → ?

Using these four rules is it possible to change MI into MU in a finite number of steps? The answer is no.

In order to prove assertions like this, it is often beneficial to look for an invariant, that is some quantity or property that doesn't change while applying the rules.

In this case, one can look at the total number of I in a string. The *invariant property* is that the number of I is never divisible by 3 which means that we can't transform the word MI into the word MU.

From *Godel, Escher, Bach*, by Douglas Hofstadter

Questions

1. Complete the symbolic transformation rules in the text.

2. Apply to the word MI the transformation process six times. They are different possibilities to do so. Try to find one that uses each rule at least once.

3. Check the invariant property in each of your previous steps.

4. Prove that if the number of I is not a multiple of 3, no transformation rule will make it a multiple of 3.

5. Deduce the answer to the puzzle.