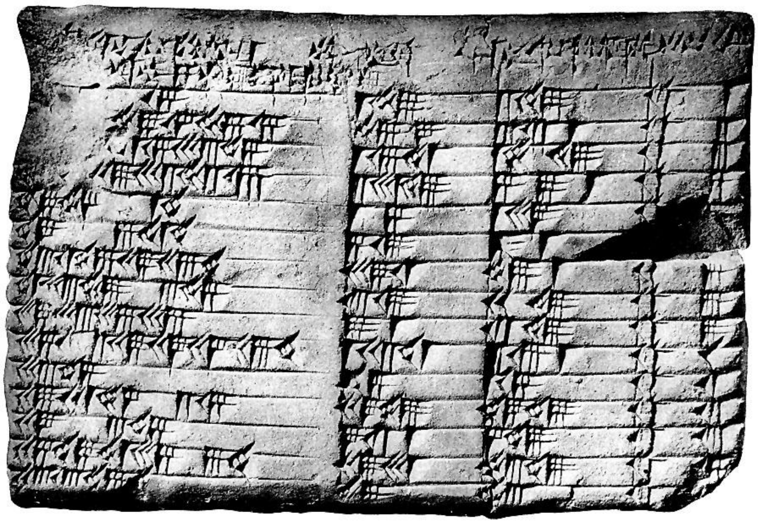
Épreuve de section européenne

Pythagorean triples

Plimpton 322 is a Babylonian clay tablet, notable as containing an example of Babylonian mathematics. This tablet, believed to have been written about 1800 BC, has a table of four columns and fifteen rows of numbers in the cuneiform script of the period.



This table lists what are now called Pythagorean triples, triplets of integers *a, b, c* satisfying $a^2 + b^2 = c^2$. For example, 3,4,5 is a Pythagorean triple, because $3^2 + 4^2 = 5^2$.

From a modern perspective, a method for constructing such triples is a significant early achievement, known before only among the Greeks. At the same time, one should recall the tablet's author was a scribe, rather than a professional mathematician; it has been suggested that one of his goals may have been to produce examples for school problems.

The Pythagorean number theorem states that if m and n are two positive integers with $m > n > 0$, then $a = 2mn$; $b = m^2 - n^2$; $c = m^2 + n^2$ is a Pythagorean triple. This result is given in Euclid's *Elements* (c. 300 B.C.).

From various sources

Questions

1. Check that 5,12,13 is a Pythagorean triple.
2. Let n be an integer greater than 1. Check that $3n$, $4n$, $5n$ is a Pythagorean triple and then prove that the set of Pythagorean triples is infinite.
3. Prove by contradiction that a Pythagorean triple can never be made of three odd numbers.
4. Apply the Pythagorean number theorem with $m = 4$ and $n = 3$. Check that you get a Pythagorean triple.
5. Prove that if m and n are two positive integers with $m > n > 0$, then $a = 2mn$; $b = m^2 - n^2$; $c = m^2 + n^2$ is a Pythagorean triple.