Gaussian integers	Season	3
	Episode	03
	Time frame	1 period

Prerequisites : Basic algebra skills.

Objectives :
- Discover complex numbers and some of their properties through Gaussian integers.
- Link geometry and algebra.

Materials :
- *Matching cards with a pair of Gaussian integers or a pair of points on a lattice.*
- *Lattice with axes and the units* 1 *and* i *(one for each student).*
- *Beamer and lesson about Gaussian integers.*

1 – Matching game with Gaussian integers and points. 15 mins

Students are handed out cards with either a pair of Gaussian integers of a pair of points on a lattice. Thay mingle to match the cards by pairs.

2 – Experimental study of the operations in $\mathbf{Z}[i]$ 25 mins

The teacher introduces the notations of the set of Gaussian integers $\mathbf{Z}[i]$ and the two operations, $+$ and $\times$.
Working in teams of two, three of four, students have to experiment on operations in the set of Gaussian integers. The aim is to find the graphical analogous to
- addition ;
- multiplication by a real number ;
- multiplication by i ;
- multiplication by a number ib.

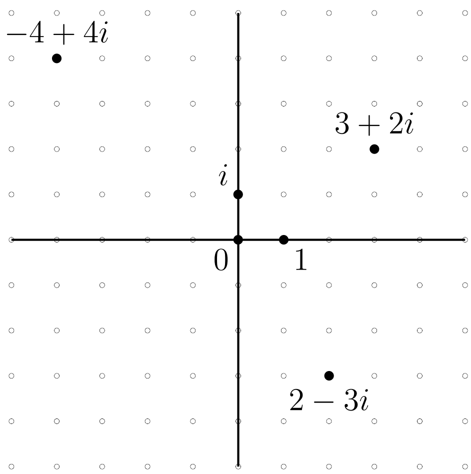
Answers are then given with a beamer. ?

Gaussian integers	Season	3
	Episode	03
	Document	Lesson

Definition 1

A *Gaussian integer* is a number of the form $a + bi$ where a and b are two integers and i is an imaginary number such that $i^2 = -1$. The set of all Gaussian integers is usually noted $\mathbf{Z}[i]$.

Every Gaussian integer is uniquely associated to a point in a square lattice. This process defines a kind of coordinate system on the lattice, with unit 1 on the horizontal axis and i on the vertical one.



The set of Gaussian integers is an extension of the set of standard integers. Addition and multiplication in the set $\mathbf{Z}[i]$ follow the same rules as addition and multiplication of integers, with the added properties that $i^2 = -1$ and that the "real parts" and "imaginary parts" cannot be mixed up in addition. We deduce easily the following properties.

Proposition 1

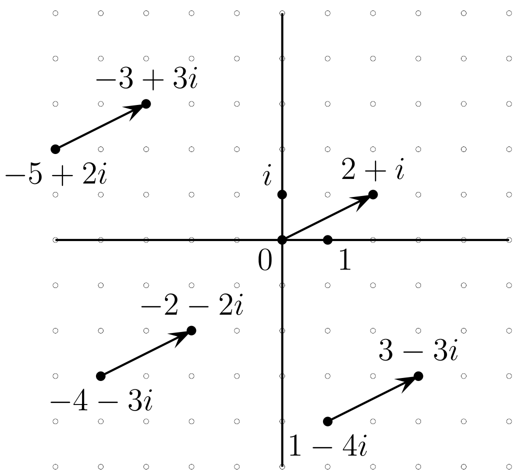
Let $a+bi$ and $c+di$ be two Gaussian integers. Their *sum* is equal to the Gaussian integer

$$(a + c) + (b + d)i$$

while their *product* is equal to

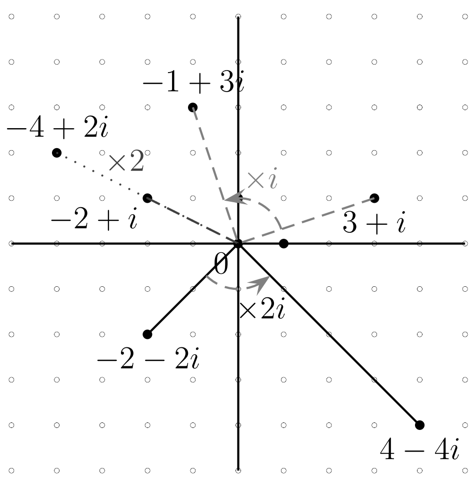
$$(ac - bd) + (ad + bc)i.$$

Operations on Gaussian integers have simple geometrical counterparts. Addition by a Gaussian integer $a + bi$ is a translation by the vector defined by the lattice points 0 and $a + bi$. The example below shows addition by $2 + i$.

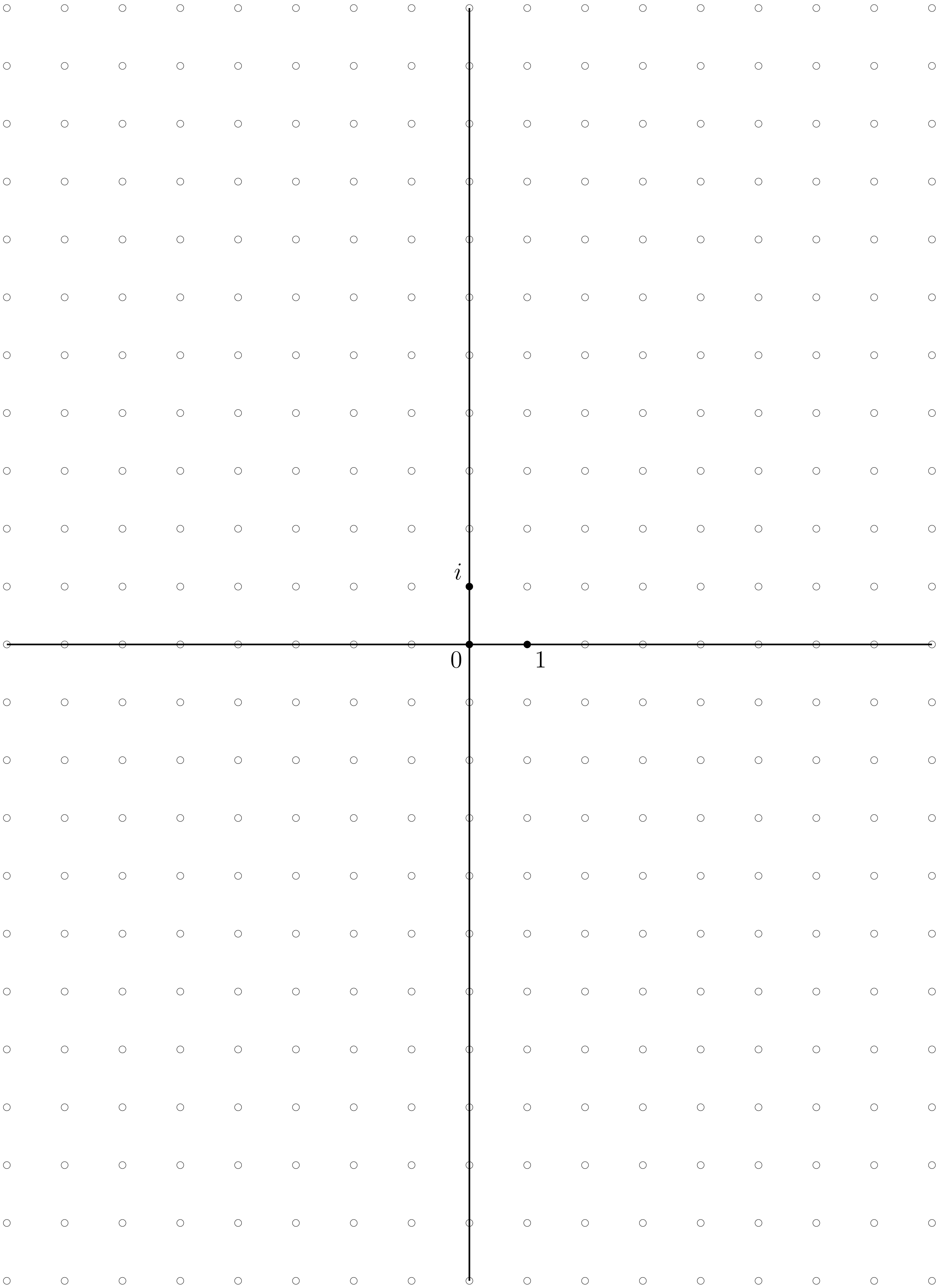


Multiplication by a Gaussian integer $a + bi$ is the composition of a rotation and an homothety around the lattice point 0 (the origin), whose angle and ratio are defined by the lattice points 0 and $a + bi$. Here are a few examples.

- Multiplication by i is a rotation of angle $\frac{\pi}{2}$ around the origin, as the distance between 0 and i is equal to 1.
- Multiplication by 2 is an homothety of ratio 2 around the origin, as the angle defined by the point 2 is equal to 0.
- Multiplication by $2i$ is the composition of an homothety of ratio 2 and a rotation of angle $\frac{\pi}{2}$, both around the origin.

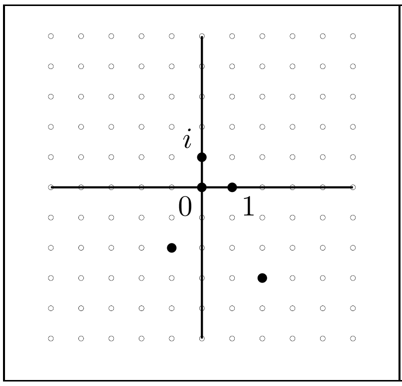


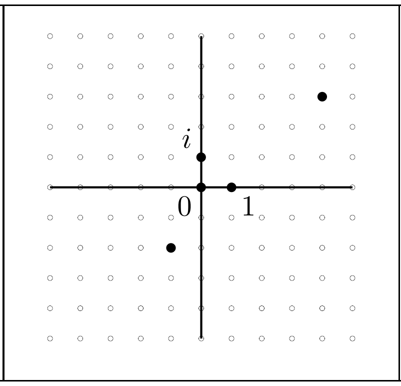
Document 1 Lattice for experiments

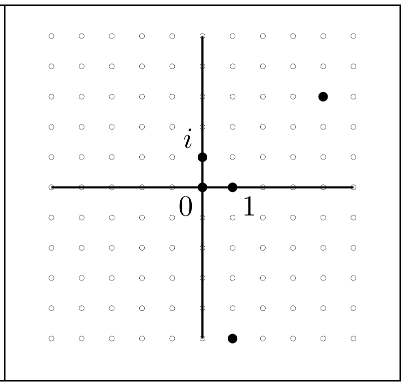


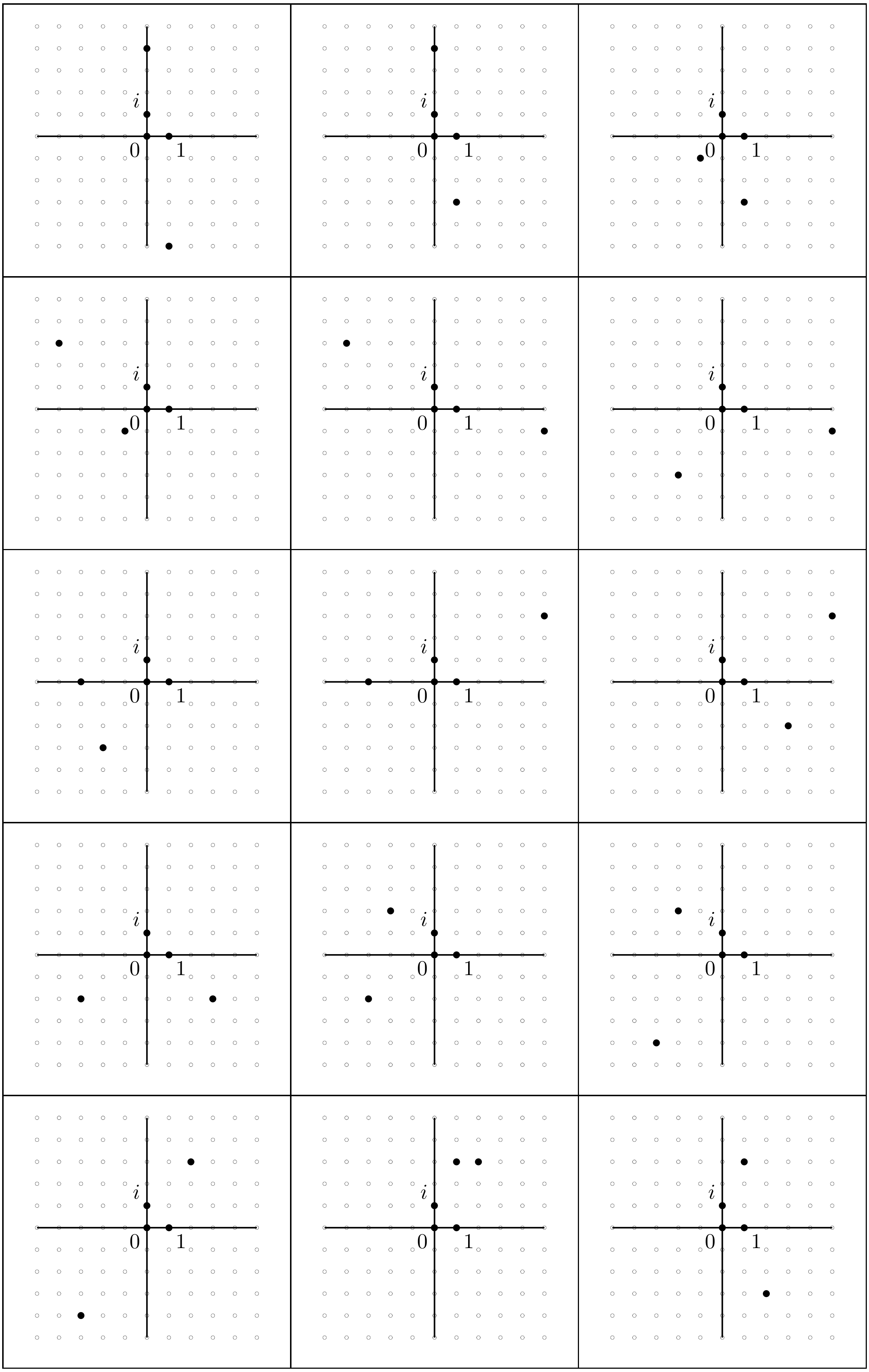
Document 2 Matching cards with Gaussian integers and points

$2-3i$ $-1-2i$	$-1-2i$ $4+3i$	$4+3i$ $1-5i$	$1-5i$ $4i$
$4i$ $1-3i$	$1-3i$ $-1-i$	$-1-i$ $-4+3i$	$-4+3i$ $5-i$
$5-i$ $-2-3i$	$-2-3i$ -3	-3 $5+3i$	$5+3i$ $3-22i$
$3-2i$ $-3-2i$	$-3-2i$ $-2+2i$	$-2+2i$ $-3-4i$	$-3-4i$ $2+3i$
$2+3i$ $1+3i$	$1+3i$ $2-3i$		








i
0
1