Constructible polygons	Season	3
	Episode	07
	Time frame	2 periods

Prerequisites : Ruler and compass rules and methods

Objectives :
● study the constructibility of regular polygons.

Materials :
● *Ruler*
● *Compas*
● *Task sheet*
● *Hints for the pentagon, 15-gon and 16-gon.*
● *Beamer*

1 – The easy ones   20 mins

Students work in pairs. They have to find the minimal number of actions needed to draw an equilateral triangle, a square, a regular hexagona, and a regular octogon. This part is marked over 10.

2 – The regular pentagon   20 mins

The second task is to construct a regular pentagon. This part is marked over 10. Progressive hints are available on demand. Every hint asked by a group takes one point of the final mark.

3 – The regular pentadecagon   15 mins

The third task is to construct a regular 15-gon. One hint may be given : use a pentagon and an equilateral triangle. This par is not marked

4 – The regular heptadecagon   20 mins

The third task is to construct a regular 17-gon. A construction protocol is given to each student and has to be carried out. The end result is also marked over 10.

5 – Constructible polygons   20 mins

Each group has to list all constructible polygons with a number of sides less than or equal to 20.
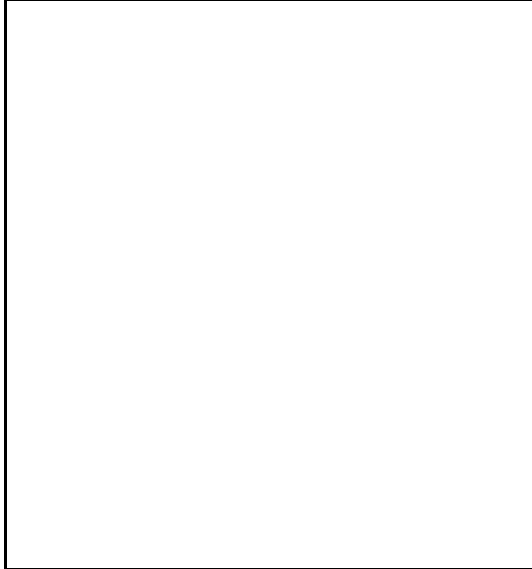
6 – Lecture : Some results about constructible polygons   15 mins

A quick history of the problem of constructible polygons, including Euclid's methods, Gauss' results and Gardner's link with the Sierpinski's binary sieve.

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	Document	Answer sheet

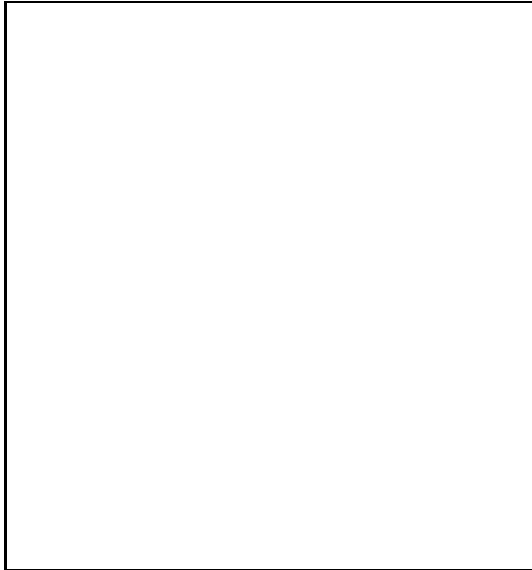# Part 1 – Construct some easy regular polygons

Equilateral triangle

Square

Regular Octogon

Regular hexagon

Construction of the pentagon

Try to find a way to construct a regular pentagon. If you don't manage, go to the teacher's desk and ask for a hint. The hints are progressive and cost 1 point over 10 each.

Construction of the pentadecagon

Try to find a way to construct a regular pentadecagon (with 15 equal sides). If you don't manage, go to the teacher's desk and ask for a hint. There is only one available hint and this part is not marked.

The heptadecagon : a construction protocol

Follow the following instructions to construct a regular heptadecagon.

1. Given an arbitrary point O, draw a circle centered on O and a horizontal diameter drawn through O.

2. Call the right end of the diameter dividing the circle into a semicircle P_1.

3. Construct the diameter perpendicular to the original diameter by finding the perpendicular bisector OB, with B at the top of the circle.

4. Construct J a quarter of the way up OB.

5. Join JP_1 and find E on line segment OP_1 so that $\angle OJE$ is a quarter of $\angle OJP_1$.

6. Find F on line OP_1, but on the other side of O, so that $\angle EJF$ is 45 degrees.

7. Construct the semicircle with diameter FP_1, on the same side as J. This semicircle cuts OB at K.

8. Draw a semicircle with center E and radius EK, on the same side as B and with both endpoints on OP_1. This cuts the line segment OP_1 at N_4.

9. Construct a line perpendicular to OP_1 through N_4. This line meets the original semicircle at P_4.

10. You now have points P_1 and P_4 of a heptadecagon. Use P_1 and P_4 to get the remaining 15 points of the heptadecagon around the original circle by constructing P_1, P_4, P_7, P_{10}, P_{13}, P_{16}, P_2 and so on.

11. Connect the adjacent points P_i for $i = 1$ to 17, forming the heptadecagon.

The constructible polygons

1. List all regular polygons with 20 or less sides that you think are constructible withe ruler and compass. Explain in a few words each construction.

2. A Fermat prime is a prime number of the form $F_n = 2^{2^n} + 1$, where n is a nonnegative integer. Compute the first five Fermat primes.

3. There is a connection between the constructible polygons and the Fermat prime. Try to find it.

Document 1   Hints for the construction of the regular pentagon

1. Draw a circle in which to inscribe the pentagon and mark the center point O.

2. Choose a point A on the circle that will serve as one vertex of the pentagon. Draw a line through O and A.

3. Construct a line perpendicular to the line OA passing through point O. Mark its intersection with one side of the circle as the point B.

4. Construct the point C as the midpoint of O and B.

5. Draw a circle centered at C through the point A. Mark its intersection with the line OB (inside the original circle) as the point D.

6. Draw a circle centered at A through the point D. Mark its intersections with the original circle as the points E and F.

7. Draw a circle centered at E through the point A. Mark its other intersection with the original circle as the point G.

8. Draw a circle centered at F through the point A. Mark its other intersection with the original circle as the point H.