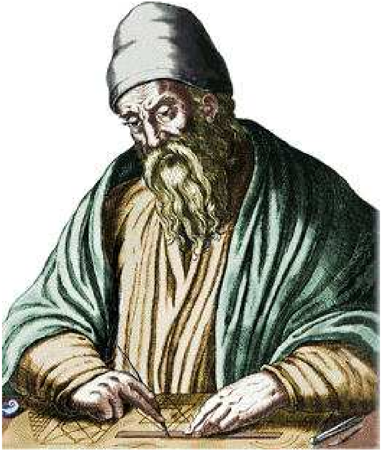
Episode 07 – Constructible polygons

European section – Season 3

Euclid

Euclid was a Greek mathematician and is often referred to as the “Father of Geometry.” He was active in Hellenistic Alexandria during the reign of Ptolemy I (323-283 BC). He knew how to inscribe a regular polygon with 3, 4, 5, 6, 8, 10, 12, 15, 16, 20, 24, 30, 32, 40, 48, 60, 64,. . . , sides.



Carl Friedrich Gauss proved the constructibility of the regular 17-gon in 1796. Five years later, he developed the theory of Gaussian periods in his *Disquisitiones Arithmeticae*. This theory allowed him to formulate a sufficient condition for the constructibility of regular polygons:

Gauss' result

A regular n-gon can be constructed with compass and straightedge if n is the product of a power of 2 and any number of distinct Fermat primes.

Gauss conjectured that this condition was also necessary. It was proved by Pierre Wantzel in 1837.

Constructible polygons

Using Gauss' result, we can decude that the regular n-gon is constructible for the following values of n :

$$3, 4, 5, 6, 8, 10, 12, 15, 16, 17, 20, 24,$$

$$30, 32, 34, 40, 48, 51, 60, 64, \ldots$$

Pascal's triangle and the Sierpinski sieve

						1						
					1		1					
				1		2		1				
			1		3		3		1			
		1		4		6		4		1		
	1		5		10		10		5		1	
1		6		15		20		15		6		1

Pascal's triangle and the Sierpinski sieve

						1						
					1		1					
				1		0		1				
			1		1		1		1			
		1		0		0		0		1		
	1		1		0		0		1		1	
1		0		1		0		1		0		1

Pascal's triangle and the Sierpinski sieve

1	**1**
1 1	**3**
1 0 1	**5**
1 1 1 1	**15**
1 0 0 0 1	**17**
1 1 0 0 1 1	**51**
1 0 1 0 1 0 1	**85**